

Note on a New Fundamental Length Scale l Instead of the Newtonian Constant G^*

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Abstract

The newly proposed entropic gravity suggests gravity as an emergent force rather than a fundamental one. In this approach, the Newtonian constant G does not play a fundamental role any more, and a new fundamental constant is required to replace its position. This request also arises from some philosophical considerations to contemplate the physical foundations for the unification of theories. We here consider the suggestion to derive G from more fundamental quantities in the presence of a new fundamental length scale l , which is suspected to originate from the structure of quantum space-time, and can be measured directly from Lorentz-violating observations. Our results are relevant to the fundamental understanding of physics, and more practically, of natural units, as well as explanations of experimental constraints in searching for Lorentz violation.

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The unification of gravitational force with the other three forces in the standard model of particle physics is a most intriguing aim of the physicist community. However, explorations in this direction have been confronted with severe difficulties, and we are still without a satisfactory solution. There have been other attempts to understand gravity as an emergent phenomenon based on quantum theory and thermodynamics [1, 2]. Recently, Verlinde proposed an appealing argument to explain gravity as an entropic force [3] from holographic perspectives [4, 5]. In Verlinde’s approach, gravity comes from the tendency of maximizing entropy in black hole physics, *i.e.*, gravity is actually an entropic force instead of a fundamental one. He conjectured that the entropy change ΔS associated with a displacement, Δx , towards a holographic screen by a particle with mass m , is

$$\Delta S = 2\pi k_B \frac{mc}{\hbar} \Delta x, \quad (1)$$

where c , k_B , and \hbar are the speed of light in vacuum, the Boltzmann constant, and the reduced Planck constant, respectively. By utilizing holographic principles and the Unruh temperature [6], Verlinde succeeded in reproducing Newton’s second law. By introducing a number of used bits on the holographic screen together with the equipartition Ansatz of mass M within the holographic screen, he also derived Newton’s law of gravity.

From a logical viewpoint, if the gravitational force represents an emergent force rather than a fundamental one, the Newtonian constant G does not play a fundamental role any more, but instead, should be considered as a deduced quantity from more basic constants. Conventionally, the gravitational constant G is one of the five “God-given” ingredients, *i.e.*, c , G , \hbar , k_B , and $1/4\pi\epsilon_0$ (where ϵ_0 is the permittivity of free space), to construct the Planck units [7]. In 1899, Planck set the above five constants as bases, and elegantly simplified recurring algebraic expressions in physics. The nontrivial non-dimensionalization has conceptually profound significance for theoretical physics. There are a number of basic quantities in this unit system, such as the Planck length $l_P \equiv \sqrt{G\hbar/c^3} \simeq 1.6 \times 10^{-35}$ m, the Planck time $t_P \equiv \sqrt{G\hbar/c^5} \simeq 5.4 \times 10^{-44}$ s, the Planck energy $E_P \equiv \sqrt{\hbar c^5/G} \simeq 2.0 \times 10^9$ J, and the Planck temperature $T_P \equiv \sqrt{\hbar c^5/Gk_B^2} \simeq 1.4 \times 10^{32}$ K [7].

Now if G is removed from the five basic ingredients, we need another fundamental quantity to complete the unit system. It can be an energy scale, or a length scale, or a time scale, and such a fundamental quantity is necessary to retrieve G as a deduced quantity. We here choose a new fundamental length scale l , following a recent suggestion [8, 9]. If l happens

to coincide with l_P , then the unit system remains intact by replacing G with l_P . Otherwise, there will be conceptual consequences on the unit system.

On the other hand, there have been many debates on the issue of fundamental units. There is a famous trialogue on the number of dimensionful units of fundamental significance among Duff, Okun, and Veneziano [10]. Okun insisted that space, time, and mass are three basic physical dimensions which can arise from special relativity (c), quantum mechanics (\hbar), and gravity (G). Among them, “The position of G is not as firm as that of c and \hbar ”, stated by Okun [10]. G , \hbar , and c form a “cube of theories”, with vertices denoting our understanding of physics related to distinguished physical realities [11]. Veneziano argued that two fundamental units of space and time, the light speed c and a string length scale λ_s , are both necessary and sufficient to be fundamental in the framework of quantum string theory and/or M-theory. However, Duff advocated that all dimensionful units are merely conversion factors, and only dimensionless quantities really matter to physical processes. It seems that the position of G as fundamental is reasonably doubted by all of the three sides in the trialogue.

It is necessary to mention that, the well-known, widely-used, and globally-adopted international system of units (abbreviated as “SI” from the French “Système international d’unités”), which possesses seven units, is not relevant to discussions here, because that they are actually related to a system of units of measurements, rather than fundamental physics.

Bearing the suspicion of gravity and G as fundamental in mind, let us analyze how G is introduced by Verlinde. For a region of mass M enclosed by a spherical surface $A = 4\pi R^2$, all information stored on A is assumed to be given by the number of bits [3]

$$N = \frac{Ac^3}{G\hbar} \equiv \frac{A}{S_P}, \quad (2)$$

where G , or equivalently the Planck area $S_P \equiv l_P^2$ [12], is introduced as a parameter in the information assignment process. By adopting the energy equipartition principle, $E = N \cdot k_B T / 2 = Mc^2$, and the well-known thermal relation, $F\Delta x = T\Delta S$, Eqs. (1) and (2) lead to Newton’s gravity law,

$$F = G \frac{Mm}{R^2}. \quad (3)$$

Hence G is identified as the gravitational constant [3]. From above, we see that the Planck area $S_P \equiv l_P^2$, or the Planck length l_P , might be considered as a more basic quantity to replace

G in the entropic gravity approach [12]. However, here the holographic bits assignment process is involved, whose detailed dynamics is yet unknown. Hence l_P as the fundamental length scale is not decisive; actually, it is quite questionable. The String Theory Committee might argue that the string length λ_s is much more fundamental. Ref. [13] hypothesized that gauge symmetry, as well as bosons, can be emergent, and suggested the ultraviolet cutoff of fermions, which might be associated with Lorentz symmetry violation, to be basic. From viewpoints of fermionic vacuum polarization, the cutoff can be as large as $10^{25}E_P$ in the presence of three families of fermions, and 10^8E_P with five families [13]. Besides, some branches of gravity society themselves are also endeavoring in the search for a more basic origin of gravity, instead of accepting it as given. Therefore, there are strong motivations for a new quantity to replace G as fundamental.

We here follow the suggestion to introduce a fundamental length scale l , which, from dimensional analysis, is proportional to l_P , *i.e.*, $l = l_P/\eta$ [8, 9]. Then from Eq. (2), we have

$$N = \frac{A}{\eta^2 l^2}. \quad (4)$$

We see that η^2 reflects the relation of the physically fundamental area, l^2 , and the unit area needed to store one bit of information from holographic perspectives.

Consequently, the conventional Newtonian constant G appears to be [8, 9]

$$G = \frac{\eta^2 l^2 c^3}{\hbar}, \quad (5)$$

where it is considered as derived from fundamental constants c , \hbar , and l . Therefore, G does not play a fundamental role as a priori any more in the entropic gravity scenario [9].

Worthy to note that, as suggested by Verlinde, after including the Unruh temperature $T = \hbar a / 2\pi c k_B$ [6], where a denotes the acceleration of motion, the second law of Newton, $F = ma$, also emerges directly [3, 12]. Further by introducing the equivalence principle, Einstein equations are also obtained [3], in a similar way with an analogy between general relativity and thermodynamics [14].

Now let us turn to our main concern on the relation between G and l . Naively speaking, η is “defined” in Eq. (4) and reflects the dynamics of information assignment onto the holographic screen. It plays a role of some unknown principles of storing information [9]. Once the procedure of digital storage onto holographic screens is well understood, it is practical to calculate η from certain holographic rules. However, nowadays, due to the imperfect

knowledge of the holographic mechanism, we cannot get its value theoretically, even through evaluation. Fortunately, it can be settled through Lorentz violating experiments [9], after we identify it to be related to the minimal structure of quantum space-time, or equivalently, quantum gravity. The indeterminate value of η influences the fundamental length scale l and the corresponding energy scale E , as well as explanations of experimental data in the search of Lorentz symmetry violation.

The search for Lorentz violation has energetically lasted for more than ten years; however, no definite deviation from special relativity is yet confirmed [15, 16]. Hence one begins to doubt the possibility and reasonableness of Lorentz violation. If the arguments above turn out to be valid, a new fundamental length scale l can be introduced; then the conventional “criterion”, *i.e.*, the Planck quantities l_P and E_P , should be modified. In the case of large η , *e.g.*, $\eta > 10^8$, most constraints from astrophysical and laboratory observations can be compatible to Lorentz symmetry violation with linearly modified terms. As another merit, the Lorentz-violating research has already validated plenty of experiments and observations to explore the relevant energy scale. Thus it appears a promising strategy to determine the fundamental length scale l through Lorentz violation, *e.g.*, time delays of high energy photons relative to low energy ones [15–23], the reaction patterns of ultra-high energy cosmic rays and TeV γ -rays with background low energy photons, and the instances of some forbidden reactions [15, 16, 20].

Conventionally, l_P represents a fundamental quantity in the search of quantum gravity. Actually, it is used as a “criterion” to justify quantum gravity phenomenons [15–25]. For example, in Ref. [22], the Fermi GBM/LAT Collaboration estimated the quantum gravity energy scale from time delays of high energy photons of a γ -ray burst, GRB 090510, to be larger than the Planck energy E_P . It was argued to rule out the linear energy dependence of Lorentz-violating modifications on the speed of light. However, if the referenced energy scale is some orders larger in magnitude than E_P , their conclusion would not be valid. In that case, the experimental implications on whether to rule out linear quantum-gravitational modifications on the light speed should be re-explained.

It is clear that $\eta \rightarrow 1$ leads to $l \rightarrow l_P$. However, according to discussions above, η is not protected to be around unit. The possibility that η is far away from unit is not theoretically forbidden from holographic viewpoints, and other theoretical considerations (see, *e.g.*, Ref. [13] for a rather large $\eta \sim 10^{25}$ in presence of three families of fermions, and

$\eta \sim 10^8$ in presence of five families of fermions). For a generic $\eta \neq 1$, we have a new length scale l for reference instead of the conventional l_P , as well as a new energy scale $E = \hbar c/l$ instead of E_P . Experimentally, the positions of \hbar and c are more firm than that of G , so we here adopt them as intact. To do comparisons more conveniently, we write the new scales in terms of conventional Planck scales,

$$l = l_P/\eta, \quad E = \eta E_P. \quad (6)$$

If $\eta = 10^8$, then $l = 10^{-8}l_P$ and $E = 10^8 E_P$. This represents an illustrated case where the space needed to store per piece of information is rather larger than the physically fundamental area l^2 , *i.e.*, $10^{16}l^2 = S_P$ for one bit. However, the situation is possible (reasonable) in reality, maybe ascribing to the poor quality of physical (foamy) areas for storing holographic information.

Now let us further discuss the physical implications of the newly introduced l . The most likely scenario to accommodate such a fundamental length scale is the space-time foam conjecture, firstly suggested by Wheeler in nineteen fifties [26]. It is a speculative extension of the space-time concepts with hypothesis of the coexistence of the matter-geometry interrelation and the uncertainty-principle-induced high-energy virtual particles at very short distances. As the length scale approaches l , the classically continuous description of space-time breaks down, and novel quantum gravity theories are needed to complete a consistent description of physical processes. Here, the proposed l can be the physically smallest scale of space-time structure. For a concrete example, l can be explained as the scale where the space-time coordinates fail to commute, as in the non-commutative quantum field theories [27].

However, another puzzle raises here immediately. Due to the well-known Lorentz-Fitzgerald contraction in the special relativity, we would expect frame-dependence of l , which threatens its “fundamentality”. There are two possibilities to remedy this problem, and both are extensively studied in the Lorentz-violating literatures; however, both are still controversial [15, 16]. The first is that, indeed, l depends on the choice of frames, and it transforms between different frames according to some laws (not necessary to obey the conventional Lorentz transformation and the symmetries of Poincaré group). When the relevant length scale is far larger than l , the transformation laws become coincident with Lorentz transformation. The “fundamentality” of l relies on the existence of a preferred frame, which

is often chosen as the frame where the cosmic microwave background is isotropic. Only in this preferred frame, observations agree on the fundamental value of l [15, 16]. The other possibility is that observations within different frames agree on the same value of l . A theoretical framework to be compatible with such an extra physical constant is realized in doubly special relativity, firstly proposed by Amelino-Camelia [24, 25, 28]. In this framework, all observers situated in different inertial frames agree on two fundamental quantities; the speed of light c and a length scale l_{DSR} . Here we identify l_{DSR} as the l discussed in the present paper. The “fundamental” meaning of l is relativistic, even better achieved conceptually. Its position is the same as the speed of light c in the special relativity; hence, l and c share the same conceptual significance in this approach.

From experimental aspects, Klinkhamer suggested four kinds of experiments to determine different physical quantities, l and η (or alternatively, G) [9]. The traditional and modern versions of the Cavendish experiment can measure the Newtonian coupling constant $G \propto \eta^2 l^2$. A complementary way to disentangle η and l comes from the mentioned Lorentz-violating research, where a separated determination of l , or alternatively E in Eq. (6), can be achieved from the modified energy-momentum dispersion relation [15–25]. The last two experiments considered in Ref. [9] are *gedanken experiments* from primordial gravitational waves and a logarithmic correction of the entropy, which can probe an isolated l , and, if proven correct and becomes practical, they can serve as consistent checks to the above two practical means.

In summary, from Verlinde’s conjecture that gravity is an emergent force rather than a fundamental one, the Newtonian constant G can no longer function as a fundamental constant. The request also merges from other ideas through contemplations on physical foundations towards unification. Then it is natural to suggest a fundamental length scale l to replace the position of G . Such an l can be explained as the smallest length scale of quantum space-time, and its value can be measured directly through searches of Lorentz violation.

In principle, l can be far away from the conventional Planck length, $l_P \simeq 1.6 \times 10^{-35}$ m. The actual value of l has consequences in many physical aspects, especially in the search of quantum gravity theories and ultimate understanding of the structure of quantum space-time. The “rescaling” of l from l_P can be understood through the detailed mechanism of assigning bits of information onto holographic areas in the entropic gravity approach. The

rescaling of the fundamental length scale of quantum space-time influences the conclusions drawn from Lorentz-violating studies. The related experiments to disentangle the involved physical quantities are also discussed, and the ones from Lorentz-violating aspects are rather promising. It also can provide new perspectives on the fundamental issue of basic space-time units, hence the whole system of “natural units”.

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- [1] See, e.g., Padmanabhan T. Thermodynamical Aspects of Gravity: New insights. Rept Prog Phys, 2010, 73: 046901 [arXiv:0911.5004 [gr-qc]], and references therein
 - [2] Padmanabhan T. Equipartition of energy in the horizon degrees of freedom and the emergence of gravity. Mod Phys Lett A, 2010, 25: 1129 [arXiv:0912.3165 [gr-qc]]
 - [3] Verlinde E P. On the origin of gravity and the laws of Newton. JHEP, 2010, 1104: 029 [arXiv:1001.0785 [hep-th]]
 - [4] 't Hooft G. Dimensional reduction in quantum gravity. 1993 [arXiv:gr-qc/9310026]
 - [5] Susskind L. The World As A Hologram. J Math Phys, 1995, 36: 6377 [arXiv:hep-th/9409089]
 - [6] Unruh W G. Notes on black hole evaporation. Phys Rev D, 1976, 14: 870
 - [7] Planck M. Über irreversible Strahlungsvorgänge. Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin, 1899, 5: 440
 - [8] Klinkhamer F R. Fundamental length scale of quantum spacetime foam. JETP Lett, 2007, 86: 73 [arXiv:gr-qc/0703009]
 - [9] Klinkhamer F R. Newtonian gravity as an entropic force: towards a derivation of G. Class Quantum Grav, 2011, 28: 125003 [arXiv:1006.2094 [hep-th]]
 - [10] Duff M J, Okun L B and Veneziano G. Trialogue on the number of fundamental constants. JHEP, 2002, 0203: 023 [arXiv:physics/0110060]
 - [11] See, e.g., Okun L B. Fundamental constants of physics. Sov Phys Usp, 1991, 34: 818

- [12] He X-G and Ma B-Q. Black Holes and Photons with Entropic Force. Chin Phys Lett, 2010, 27: 070402 [arXiv:1003.1625 [hep-th]]
- [13] Klinkhamer F R and Volovik G E. Merging gauge coupling constants without grand unification. Pisma Zh Eksp Teor Fiz, 2005, 81: 683 [JETP Lett, 2005, 81: 551] [arXiv:hep-ph/0505033]
- [14] Jacobson T. Thermodynamics of space-time: The Einstein equation of state. Phys Rev Lett, 1995, 75: 1260 [arXiv:gr-qc/9504004]
- [15] Mattingly D. Modern tests of Lorentz invariance. Living Rev Rel, 2005, 8: 5 [arXiv:gr-qc/0502097]
- [16] Amelino-Camelia G, Quantum gravity phenomenology. 2008 [arXiv:0806.0339 [gr-qc]]
- [17] Amelino-Camelia G, Ellis J R, Mavromatos N E, Nanopoulos D V and Sarkar S. Potential Sensitivity of Gamma-Ray Burster Observations to Wave Dispersion in Vacuo. Nature, 1998, 393: 763 [arXiv:astro-ph/9712103]
- [18] Xiao Z and Ma B-Q. Constraints on Lorentz invariance violation from gamma-ray burst GRB090510. Phys Rev D, 2009, 80: 116005 [arXiv:0909.4927 [hep-ph]]
- [19] Shao L, Xiao Z and Ma B-Q. Lorentz violation from cosmological objects with very high energy photon emissions. Astropart Phys, 2010, 33: 312 [arXiv:0911.2276 [hep-ph]]
- [20] Shao L and Ma B-Q. Lorentz violation effects on astrophysical propagation of very high energy photons. Mod Phys Lett A, 2010, 25: 3251 [arXiv:1007.2269 [hep-ph]]
- [21] Xiao Z, Shao L and Ma B-Q. “Eikonal equation of the Lorentz-violating Maxwell theory,” Eur Phys J C, 2010, 70: 1153 [arXiv:1011.5074 [hep-th]]
- [22] Abdo A A et al. Testing Einstein’s special relativity with Fermi’s short hard gamma-ray burst GRB090510. Nature, 2009, 462: 331 [arXiv:0908.1832 [astro-ph.HE]]
- [23] Shao L and Ma B-Q. “Lorentz violation induced vacuum birefringence and its astrophysical consequences,” Phys Rev D, 2011, 83: 127702 [arXiv:1104.4438 [astro-ph.HE]]
- [24] Amelino-Camelia G. Relativity in space-times with short-distance structure governed by an observer-independent (Planckian) length scale. Int J Mod Phys D, 2002, 11: 35 [arXiv:gr-qc/0012051]
- [25] Amelino-Camelia G. Doubly-Special Relativity: First Results and Key Open Problems. Int J Mod Phys D, 2002, 11: 1643 [arXiv:gr-qc/0210063]
- [26] Wheeler J A. On the Nature of quantum geometrodynamics. Annals Phys, 1957, 2: 604

- [27] Douglas M R and Nekrasov N A. Noncommutative field theory. *Rev Mod Phys*, 2001, 73: 977
[arXiv:hep-th/0106048]
- [28] Zhang X, Shao L and Ma B-Q. Photon Gas Thermodynamics in Doubly Special Relativity.
Astropart Phys, 2011, 34: 840 [arXiv:1102.2613] and references therein